

Coherent gamma photon generation in a Bose-Einstein condensate of ^{135m}Cs

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We have identified a mechanism of collective nuclear de-excitation in a Bose-Einstein condensate of ^{135}Cs atoms in their isomeric states, ^{135m}Cs , suitable for the generation of coherent gamma photons. The process described here does not correspond to single-pass amplification, which cannot occur in atomic systems due to the large shift between absorption and emission lines, nor does it require the large densities associated to standard Dicke super-radiance. It thus overcomes the limitations that have been hindering the generation of coherent gamma rays in many systems. Therefore, we propose an approach for generation of coherent gamma rays, which relies on a combination of well established techniques of nuclear and atomic physics, and can be realized with currently available technology.

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The possibility to realize a gamma-ray laser has been an active field of research since the very first observation of lasing in the visible [1]. In fact, possible applications range from fundamental and applied physics to the bio-medical field, the energy industry and the security sector. Numerous different mechanisms have been proposed for the generation of coherent gamma radiation, such as stimulated γ emission from an ensemble of ^{229m}Th isomeric nuclei in a host crystal [2] and annihilation of positronium in a Bose-Einstein condensate [3, 4], to name a few.

In this work we propose coherent γ photon generation using a Bose-Einstein condensate (BEC) of ^{135m}Cs isomers. The use of ultra-cold atoms is attractive as it allows one to overcome two fundamental problems which have hindered the realization of a nuclear gamma-ray laser: the accumulation of a large number of isomeric nuclei, and the reduction of the gamma-ray emission linewidth. However, it is not obvious which mechanisms could lead to coherent gamma ray production in such a system. In fact, single pass amplification is inhibited by the difference in absorption and emission wavelengths, due to the large recoil associated with nuclear emission. Furthermore, Dicke super-radiance, a major candidate for γ generation [5], cannot occur as the required density is not achievable in dilute atomic BECs produced via standard techniques. In this Letter, we identify a collective de-excitation of atomic nuclei in a BEC of ^{135m}Cs isomers leading to the generation of coherent gamma rays and we demonstrate that it can be triggered also at low atomic densities. This overcomes the limitations of single-pass amplification and Dicke super-radiance.

The system proposed here for gamma ray coherent emission, unlike other approaches, has the significant advantage that it can be realized using a combination of established nuclear and atomic physics techniques. ^{135m}Cs beams can be produced by proton-induced fission of ac-

tinides [6]. After neutralisation [7], laser cooling and trapping, the evaporation to condensation should proceed as for the stable ^{133}Cs .

The nuclear system of interest is sketched in Fig. 1, which consists of an isomeric state $J^\pi = 19/2^-$ decaying to $11/2^+$ via an M4 γ transition at 846.1 keV. The half-life is $T_{1/2} = 53$ min. The intermediate state rapidly decays to the $7/2^+$ state, which corresponds to the ^{135}Cs ground state.

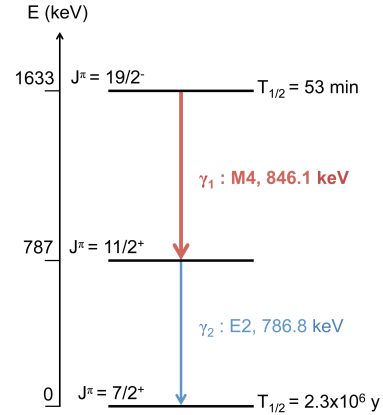


FIG. 1. De-excitation scheme of the ^{135m}Cs $J^\pi = 19/2^-$ isomeric state. A first decay to $11/2^+$ occurs via an M4 γ transition at 846.1 keV, followed by a decay to the $7/2^+$ ^{135}Cs ground state via an E2 transition. The ^{135}Cs ground state has a half-life of 2.3×10^6 y, thus its decay can be ignored for the purpose of the present study.

The Hamiltonian H of the system can be written as the sum of three terms:

$$H = H_A + H_\gamma + H_{A\gamma} . \quad (1)$$

H_A is the Hamiltonian of the free cesium atoms and

it is defined, as in the following, in the laboratory reference frame. For the analysis of gamma ray generation, only the nuclear excited state $J^\pi = 19/2^-$ and the intermediate state $11/2^+$ are considered, with the energy separation indicated by $\hbar\omega_0 = 846.1$ keV.

By introducing second quantization operators, the atomic Hamiltonian H_A can be written as:

$$H_A = \int \left[\left(\frac{p^2}{2m} + \hbar\omega_0 \right) \Theta_{\mathbf{p}}^\dagger \Theta_{\mathbf{p}} + \left(\frac{p^2}{2m} \right) X_{\mathbf{p}}^\dagger X_{\mathbf{p}} \right] d\phi_{\mathbf{p}} . \quad (2)$$

$\Theta_{\mathbf{p}}$ is the annihilation operator for an isomer in the excited state $J^\pi = 19/2^-$, with momentum \mathbf{p} , and $X_{\mathbf{p}}$ is the annihilation operator for the short-living intermediate state $11/2^+$, with momentum \mathbf{p} . Integration is over the phase-space element $d\phi_{\mathbf{p}} = \mathcal{V}/(2\pi\hbar)^3 d^3\mathbf{p}$, with \mathcal{V} the quantization volume.

The Hamiltonian of the photonic field H_γ is written as:

$$H_\gamma = \sum_{\zeta} \int d\phi_{\mathbf{k}} \hbar\omega(\mathbf{k}) c_{\mathbf{k},\zeta}^\dagger c_{\mathbf{k},\zeta} , \quad (3)$$

where $c_{\mathbf{k},\zeta}$ is the annihilation operator of a photon of momentum \mathbf{k} and helicity ζ ($\zeta = \pm 1$), with the photonic dispersion relation $\omega = c|\mathbf{k}|$. The integral is over the phase-space element $d\phi_{\mathbf{k}} = \mathcal{V}/(2\pi)^3 d^3\mathbf{k}$.

Lastly, the Hamiltonian $H_{A\gamma}$

$$H_{A\gamma} = \sum_{\zeta} \int d\phi_{\mathbf{k}} d\phi_{\mathbf{p}} \left[\mathcal{M}_{\zeta}(\mathbf{k}, \mathbf{p}) c_{\mathbf{k},\zeta}^\dagger \Theta_{\mathbf{p}} X_{\mathbf{p}-\hbar\mathbf{k}}^\dagger + \mathcal{M}_{\zeta}^*(\mathbf{k}, \mathbf{p}) c_{\mathbf{k},\zeta} \Theta_{\mathbf{p}}^\dagger X_{\mathbf{p}-\hbar\mathbf{k}} \right] \quad (4)$$

describes the interaction between the isomer and the photonic field. $\mathcal{M}_{\zeta}(\mathbf{k}, \mathbf{p})$ is the amplitude of the decay of an isomer with momentum \mathbf{p} into a photon of momentum $\hbar\mathbf{k}$ and an atom in the nuclear state $11/2^+$ with momentum $\mathbf{p} - \hbar\mathbf{k}$. In the present case, we are interested in isomers initially at rest, so we will assume $\mathbf{p} = \mathbf{0}$ in the following.

The matrix element for the magnetic operator M4 is [8]:

$$\mathcal{M}_{\zeta}(\mathbf{k}, \mathbf{0}) = -\sqrt{\frac{2\pi\hbar c}{V k}} \zeta \sum_M \langle J_1 M_1 | T_{LM} | J_2 M_2 \rangle \mathcal{D}_{M\zeta}^{L*}(R) , \quad (5)$$

with $L = 4$. $\mathcal{D}_{M\zeta}^L(R)$ is the Wigner D-matrix for a rotation R with Euler angles (α, β, γ) which takes the z -axis to the direction of \mathbf{k} . It is noteworthy that only $|\mathcal{M}_{\zeta}(\mathbf{k}, \mathbf{0})|^2$ enters the relevant expressions in the following. After some algebra [9], $|\mathcal{M}_{\zeta}(\mathbf{k}, \mathbf{0})|^2$ can be written in terms of the transition mean life $\tau = T_{1/2}/\ln 2$ and the Legendre polynomial $P_K(\cos \beta)$ of order K ($\beta \in [0; \pi]$):

$$|\mathcal{M}_{\zeta}(\mathbf{k}, \mathbf{0})|^2 = \frac{\pi\hbar^2 c}{2V k^2 \tau} (2L+1) \langle J_2 M_2 L \Delta M | J_1 M_1 \rangle^2 (-1)^{\Delta M - \zeta} \sum_{K=0}^{2L} \langle L - \Delta M L \Delta M | K 0 \rangle \langle L - \zeta L \zeta | K 0 \rangle P_K(\cos \beta) , \quad (6)$$

where $\langle j_2 m_2 j m | j_1 m_1 \rangle$ indicates the Clebsch-Gordan coefficients and $\Delta M = M_1 - M_2$.

As initial state of the system, we assume that the photonic field is in the vacuum state, and the atoms are in a BEC of the nuclear excited state $J^\pi = 19/2^-$. We also assume that there are no atoms in the intermediate short-lived state, $11/2^+$, as well as in the $7/2^+$ state. The latter assumption is perfectly justified, as in cold atom experiments it is possible to select which state to trap, given the isomeric shift, i.e. the difference in frequency of the D_2 line atomic transitions, relevant for laser cooling

and trapping, between the atoms in the nuclear excited state and in the ground state.

The system dynamics can be conveniently analyzed in the Heisenberg representation. The evolution of an operator \hat{L} is determined by:

$$i\hbar \frac{\partial \hat{L}}{\partial t} = [\hat{L}, H] . \quad (7)$$

We now introduce the operators

$\tilde{c}_{\mathbf{k},\zeta} = c_{\mathbf{k},\zeta} \exp[i\omega(\mathbf{k})t]$, $\tilde{\Theta}_{\mathbf{q}} = \Theta_{\mathbf{q}} \exp[i(\frac{q^2}{2m\hbar} + \omega_0)t]$, $\tilde{X}_{\mathbf{q}} = X_{\mathbf{q}} \exp[i\frac{q^2}{2m\hbar}t]$. The equations of motion for the new operators are [9]:

$$i\hbar\dot{\tilde{c}}_{\mathbf{k},\zeta} = \int d\phi_{\mathbf{q}} \mathcal{M}_{\zeta}(\mathbf{k}, \mathbf{q}) \tilde{\Theta}_{\mathbf{q}} \tilde{X}_{\mathbf{q}-\hbar\mathbf{k}}^+ \exp \left[-i \left(-\omega(\mathbf{k}) + \frac{q^2}{2m\hbar} - \frac{(\mathbf{q}-\hbar\mathbf{k})^2}{2m\hbar} + \omega_0 \right) t \right] \quad (8a)$$

$$i\hbar\dot{\tilde{\Theta}}_{\mathbf{q}} = \sum_{\zeta} \int d\phi_{\mathbf{k}} \mathcal{M}_{\zeta}^*(\mathbf{k}, \mathbf{q}) \tilde{c}_{\mathbf{k},\zeta} \tilde{X}_{\mathbf{q}-\hbar\mathbf{k}} \exp \left[-i \left(\omega(\mathbf{q}) - \omega_0 + \frac{(\mathbf{q}-\hbar\mathbf{k})^2}{2m\hbar} - \frac{q^2}{2m\hbar} \right) t \right] \quad (8b)$$

$$i\hbar\dot{\tilde{X}}_{\mathbf{q}} = \sum_{\zeta} \int d\phi_{\mathbf{k}} \mathcal{M}_{\zeta}(\mathbf{k}, \mathbf{q} + \hbar\mathbf{k}) \tilde{c}_{\mathbf{k},\zeta}^+ \tilde{\Theta}_{\mathbf{q}+\hbar\mathbf{k}} \exp \left[-i \left(-\omega(\mathbf{q}) + \omega_0 - \frac{q^2}{2m\hbar} + \frac{(\mathbf{q}+\hbar\mathbf{k})^2}{2m\hbar} \right) t \right] \quad (8c)$$

A pure BEC of the isomers is assumed as an initial state, therefore it is appropriate to replace the operator $\tilde{\Theta}_{\mathbf{q}}$ with c-numbers by implementing the Bogoliubov approximation:

$$\tilde{\Theta}_{\mathbf{q}} = \sqrt{n_0} \frac{(2\pi\hbar)^3}{V^{1/2}} \delta(\mathbf{q}) , \quad (9)$$

where n_0 is the volume density of the isomeric BEC. By using Eq. 9, the evolution equations reduce to:

$$i\hbar\dot{\tilde{c}}_{\mathbf{k},\zeta} = \sqrt{n_0} \xi_{\zeta}(\mathbf{k}, \mathbf{0}) \tilde{X}_{-\hbar\mathbf{k}}^+ \exp[+i\Delta_{\mathbf{k}}t] \quad (10a)$$

$$i\hbar\dot{\tilde{X}}_{\hbar\mathbf{k}} = \sqrt{n_0} \sum_{\zeta} \xi_{\zeta}(-\mathbf{k}, \mathbf{0}) \tilde{c}_{-\mathbf{k},\zeta}^+ \exp[i\Delta_{-\mathbf{k}}t] \quad (10b)$$

where $\xi_{\pm}(\mathbf{k}, 0) \equiv \sqrt{V} \mathcal{M}_{\pm}(\mathbf{k}, 0)$ and:

$$N_{\mathbf{k}}(t) = N_- + N_+ = \frac{2n_0}{\hbar^2(\delta_0^2 + \delta_1^2)} (|\xi_{-}(\mathbf{k}, \mathbf{0})|^2 + |\xi_{+}(\mathbf{k}, \mathbf{0})|^2) [-\cos(\delta_0 t) + \cosh(\delta_1 t)] , \quad (14)$$

where

$$\delta_0 + i\delta_1 = \frac{1}{\hbar} \sqrt{\hbar^2 \Delta_{\mathbf{k}}^2 - 4n_0 (|\xi_{-}(\mathbf{k}, \mathbf{0})|^2 + |\xi_{+}(\mathbf{k}, \mathbf{0})|^2)} \quad (15)$$

which produces an exponential emission rate for sufficiently large n_0 . The condition for exponential growth of $N_{\mathbf{k}}$ is given by $\delta_1 \neq 0$, which requires:

$$\hbar^2 \Delta_{\mathbf{k}}^2 - 4n_0 (|\xi_{-}(\mathbf{k}, 0)|^2 + |\xi_{+}(\mathbf{k}, 0)|^2) < 0 . \quad (16)$$

Thus, exponential photonic gain is observed in the interval of frequencies (see also inset of Fig. 2):

$$-\frac{2\sqrt{n_0}}{\hbar} \sqrt{\sum_{\zeta} |\xi_{\zeta}(\mathbf{k}, \mathbf{0})|^2} < \Delta_{\mathbf{k}} < \frac{2\sqrt{n_0}}{\hbar} \sqrt{\sum_{\zeta} |\xi_{\zeta}(\mathbf{k}, \mathbf{0})|^2} . \quad (17)$$

$$\Delta_{\pm\mathbf{k}} = \omega(\pm\mathbf{k}) + \frac{\hbar(\pm\mathbf{k})^2}{2m} - \omega_0 . \quad (11)$$

In Eq. 11 the opposite linear momenta $\hbar\mathbf{k}$ and $-\hbar\mathbf{k}$ of the photon and the ground state of the M4 transition are made explicit.

From Eqs. 10, we derive:

$$\hbar\ddot{\tilde{c}}_{\mathbf{k},\zeta} - i\hbar\Delta_{\mathbf{k}}\dot{\tilde{c}}_{\mathbf{k},\zeta} - \frac{1}{\hbar}n_0\xi_{\zeta}(\mathbf{k}, \mathbf{0}) \sum_{\zeta'} \xi_{\zeta'}^*(\mathbf{k}, \mathbf{0}) \tilde{c}_{\mathbf{k},\zeta'} = 0 . \quad (12)$$

Also from Eq. (10a) we can derive the initial condition for $\dot{\tilde{c}}_{\mathbf{k},\zeta}$:

$$\dot{\tilde{c}}_{\mathbf{k},\zeta}(0) = -\frac{i}{\hbar} \sqrt{n_0 V} \mathcal{M}_{\zeta}(\mathbf{k}, \mathbf{0}) \tilde{X}_{-\hbar\mathbf{k}}^+(0) \quad (13)$$

From the above equations, we calculate the total number of photons $N_{\mathbf{k}}(t)$ emitted in the \mathbf{k} -mode as:

At the center emission frequency $\Delta_{\mathbf{k}} = 0$, i.e. for emission frequency $\omega = \omega_0 - \omega_R$ with $\omega_R = \hbar k^2/(2m)$ the recoil frequency, the gain parameter is:

$$\delta_1 \Big|_{\Delta_{\mathbf{k}}=0} = \frac{2\sqrt{n_0}}{\hbar} \sqrt{\sum_{\zeta} |\xi_{\zeta}(\mathbf{k}, \mathbf{0})|^2} . \quad (18)$$

δ_1 exhibits a $\sqrt{n_0}$ dependence, and it is non-zero also at low atomic densities, such as those currently attainable in an atomic BEC.

The gain parameter depends on the initial and final M -states, as well as on the emission angle β , which is the Euler angle between the z -axis and the direction of photons emission, i. e. $\hat{\mathbf{k}}$. We thus consider the gain parameter averaged over the initial M -states and summed over the final ones, which becomes independent of β . Its

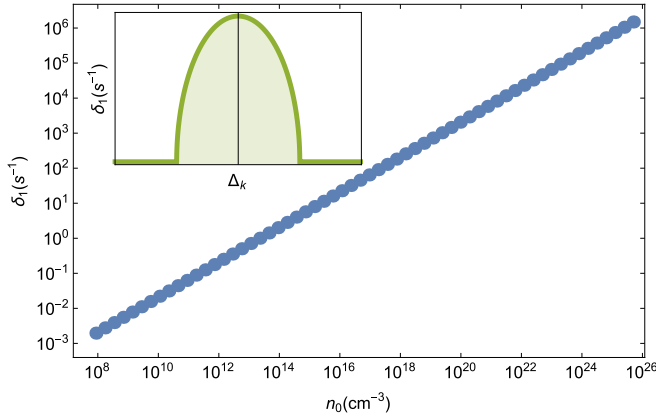


FIG. 2. Gain parameter, averaged over the initial M_1 states and summed over the final ones, as a function of the atomic density n_0 . Inset: Gain parameter lineshape, with the average gain parameter plotted as a function of Δ_k , evidencing collective emission in the interval defined by Eq. (17).

dependence on the atomic density n_0 is reported in Fig. 2, as well as a plot of the gain lineshape.

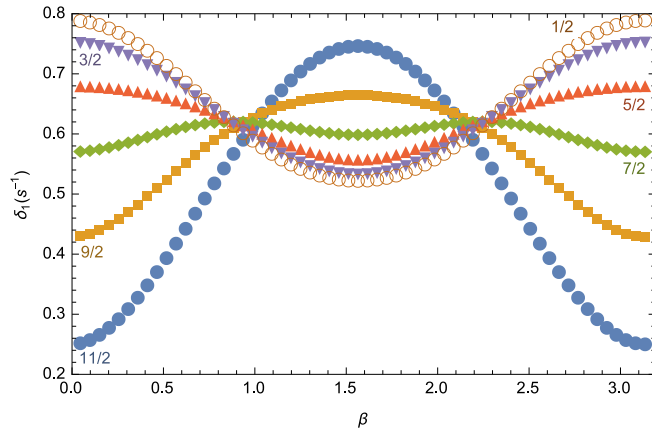


FIG. 3. Gain parameter, averaged over the initial M states, as a function of the emission angle β , for different final M -states and for an atomic density $n_0 = 10^{14} \text{ cm}^{-3}$.

Finally, in order to investigate the angular distribution of the coherent gamma emission, in Fig. 3 the dependence of the gain parameter on β is displayed for an average over the initial state M_1 and a specific choice of the final state M_2 .

Our results demonstrate that a mechanism of collec-

tive de-excitation occurs for a BEC of ^{135m}Cs isomers. The collective nature of the phenomenon is highlighted by the exponential dependence of the number of emitted photons with respect to the initial isomer density. The collective de-excitation relies on the coherence of the condensate, and occurs at densities much lower than those required by the standard Dicke super-radiance. The identified mechanism of collective de-excitation provides a promising route to the generation of coherent gamma radiation. In fact, the process relies on available technology. ^{135m}Cs ion beams can be generated by proton-induced fission of actinides. Afterwards, laser cooling and trapping can proceed as well established for ^{133}Cs and some of its isotopes [10]. The long lifetime of ^{135m}Cs allows for evaporation and creation of a BEC in an optical trap, along the lines of the procedure for stable cesium. As the collisional properties of ultra-cold ^{135m}Cs are not known, it is not possible to give an accurate estimate of the expected size of the BEC, and hence of the intensity of the gamma ray burst. However, the present results indicate that coherent emission would occur over a broad range of BEC size, thus demonstrating the validity of the proposed approach for coherent gamma ray generation.

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